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Validation of the Thermal Equilibrium Assumption in Periodic Natural Convection in Porous Domains

A. F. Khadrawi,[1] M. S. Tahat,[1] and M. A. Al-Nimr[2,3]

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The validity of the local thermal equilibrium assumption in the periodic free convection channel flow is investigated analytically. Two cases are considered where in the first case transverse conduction in the solid domain is included while in the second case transverse conduction in the fluid domain is included. The periodic disturbance in the free convection flow is due to a periodic thermal disturbance imposed on the channel walls. The Darcy–Brinkman model is used to model the flow inside the porous domain. It is found that four dimensionless parameters control the local thermal equilibrium assumption in the first case and five parameters control the local equilibrium assumption in the second case. The criteria that secure the validity of the local thermal equilibrium assumption are derived.

KEY WORDS: Darcy–Brinkman model; periodic natural convection; porous domain; thermal equilibrium assumption; validation criteria.

1. INTRODUCTION

During the last few decades, convection heat transfer in porous media has been extensively investigated. Two models are adopted to describe the thermal behavior of porous systems. These are the so-called single-phase and two-phase models [1]. The main distinction between these models is that local thermal equilibrium is assumed in the single-phase model while no such assumption is made in the two-phase model. Therefore, the single-phase model yields only one energy equation, whereas in the two-phase

[1] Mechanical Engineering Department, Al-Balqa' Applied University, Al-Salt, Jordan.
[2] Mechanical Engineering Department, Jordan University of Science and Technology, P. O. Box 3030, Irbid 22110, Jordan.
[3] To whom correspondence should be addressed. E-mail: malnimr@just.edu.jo

model there are two governing energy equations. In the two-phase model, each energy equation contains a fluid-to-solid heat transfer term.

The concept of local thermal equilibrium has been widely used in modeling transport phenomena in porous media [2–4]. Very limited investigations in the literature have included the local thermal equilibrium assumption [5–9]. Comparatively, fewer investigations have presented a comparison between both models [10–13]. All previous investigations are related only to certain cases and applications and no general result has been determined. There are many models with the conservation of momentum equation used to describe the fluid flow in porous medium: the Darcian model [6, 7, 10, 14] and the non-Darcian extensions models [2, 3, 7]. Both of these models are widely used in the literature.

The local thermal equilibrium assumption in transient forced convection porous channel flow has been investigated analytically [5, 10]. This is accomplished by focusing on the operating conditions required for both the solid and fluid domains to approximately attain the same temperature, and as a result, the local thermal equilibrium assumption is tested and confirmed.

The aim of the present study is to investigate the local thermal equilibrium assumption in the periodic free convection flow in a vertical open-ended porous channel. The fluctuations in the thermal and hydrodynamic behaviors of the problem are due to the periodic thermal disturbance imposed on the channel walls. The Darcy–Brinkman model is used to describe the hydrodynamic behavior of the free convection channel flow.

2. ANALYSIS

Consider the problem of periodic free convection fluid flow in an open-ended vertical porous channel. The fluctuations in the hydrodynamic and thermal behaviors of the channel are due to the harmonic fluctuations in the wall temperature of the channel. Referring to Fig. 1 and using the dimensionless parameters given in the nomenclature, the momentum and energy equations are given as [7]

Case I: Neglecting conduction in the fluid domain:

$$-C_a \frac{\partial U}{\partial \tau} + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{Da} - \theta_f = 0 \tag{1}$$

$$\frac{\partial \theta_s}{\partial \tau} = \frac{\partial^2 \theta_s}{\partial Y^2} + H\left(\theta_f - \theta_s\right) \tag{2}$$

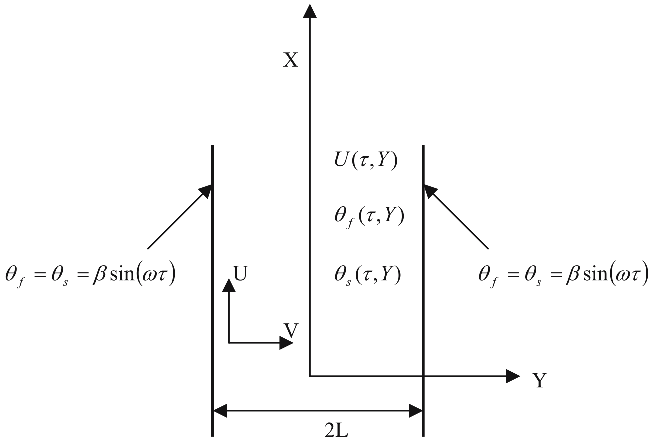

Fig. 1. Schematic diagram of the problem under consideration.

$$\frac{\partial \theta_f}{\partial \tau} = CH\left(\theta_s - \theta_f\right) \tag{3}$$

Case II: Neglecting conduction in the solid domain:

$$-C_a \frac{\partial U}{\partial \tau} + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{Da} - \theta_f = 0 \tag{4}$$

$$\frac{\partial \theta_s}{\partial \tau} = H\left(\theta_f - \theta_s\right) \tag{5}$$

$$\alpha \frac{\partial \theta_f}{\partial \tau} = \frac{\partial^2 \theta_f}{\partial Y^2} + Hr_k\left(\theta_s - \theta_f\right) \tag{6}$$

where $U = U(\tau, Y), \theta_f = \theta_f(\tau, Y), \theta_s = \theta_s(\tau, Y)$

Equations (1)–(3) or (4)–(6) assume the following initial and boundary conditions:

$$U(0, Y) = \theta_f(0, Y) = \theta_s(0, Y) = 0.0$$

$$\frac{\partial U}{\partial Y}(\tau, 0) = \frac{\partial \theta_f}{\partial Y}(\tau, 0) = \frac{\partial \theta_s}{\partial Y}(\tau, 0) = 0.0 \tag{7}$$

$$U(\tau, 1) = 0.0$$

$$\theta_f(\tau, 1) = \theta_s(\tau, 1) = \beta \sin(\omega\tau) = \beta \,\mathrm{Im}(e^{i\omega\tau})$$

In Eqs. (1)–(7), subscripts f and s refer to the fluid and solid domains, respectively, β stands for the relative amplitude of oscillations, and "Im" stands for the imaginary part of the term inside the parentheses. Also, axial thermal diffusion is not included in Eqs. (2) and (3) or (5) and (6). Axial thermal diffusion is insignificant compared to transverse diffusion and may be neglected in a channel having a low width-to-length aspect ratio. Also, axial thermal diffusion may be neglected for gases. The focus in the present study is on the main features of the local thermal equilibrium criterion, which are not affected by excluding thermal diffusion and thermal dispersion effects.

As mentioned previously, transverse conduction is neglected from the fluid energy equation for Case I and from the solid matrix energy equation for Case II. There are many physical and mathematical justifications to do so. There are many physical applications, which involve low thermal conductivity fluids (such as gases) which flow into a high thermal conductivity porous solid matrix. In these applications, thermal diffusion (conduction) in the transverse direction of the fluid domain may be neglected, especially when the porosity (void fraction) is small. A small void fraction implies that the low thermal conductivity fluid almost occupies discrete regions which are not in good thermal contact (because there is very little fluid due to the small void fraction) and as a result, transverse thermal conduction in the fluid domain is insignificant. This is true, especially in channels that have a relatively large width (L) since transverse conduction is proportional to $(1/L^2)$. On the other hand, transverse conduction may be neglected in the solid domain (as in Case II) in applications involving solids with a low thermal conductivity especially when the void fraction is large and in channels having a large width. In these applications, the solid domain consists of discrete low thermal conductivity regions, which are not in good thermal contact. In these applications, the transverse conduction in the solid domain is insignificant.

The main objective of the present work is to investigate the conditions for which the use of the two-temperature model (non-local equilibrium) is a necessity. Neglecting transverse thermal diffusion from the fluid or solid domains will not alter the main finding of the work. Also, consideration of the two separate cases (I and II) makes it possible to distinguish between the separate effects of transverse conduction in solid and fluid domains.

Another physical justification for neglecting transverse conduction in one of the two domains relies on the fact that neglecting transverse conduction presents the most severe conditions under which the use of the two-temperature model is a necessity. Consider a certain location that has a large difference between the hot solid and cold fluid temperatures. This high temperature difference implies that the use of the two-temperature model is a necessity. Now, allowing for the transverse conduction to take place within the solid domain will draw more thermal energy from the solid domain to other colder locations. This lowers the temperature difference between the fluid and solid domains and makes the conditions less severe for the two-temperature model. In other words, one may think that the use of the one-temperature model (local thermal equilibrium model) is possible but this is not always the case, especially when taking into account the fact that the two-temperature model is more general and accurate than the one-temperature model.

Also, we have a mathematical justification to consider two separate cases. If transverse conduction is considered simultaneously in both domains, we will get two coupled second-order differential equations. Decoupling these two differential equations yields a fourth-order differential equation with complex coefficients. The solution of such an equation is presented in terms of four roots obtained from the characteristic equation corresponding to this differential equation. Finding an analytical, closed-form solution in this case is not possible because it is difficult to get closed-form expressions for the four roots since they are expressed in the complex domain. In general, there is no analytical method that gives the four roots of a fourth-order equation in closed form.

It is clear from the momentum equations, Eqs. (1) and (4), that what is causing the free (natural) convection flow is the fluid temperature θ_f and not $\frac{\partial \theta_f}{\partial Y}$. The fluctuating heating source heats both fluid and solid domains through the wall, and then convective currents are initiated within the heated fluid due to the buoyancy effects. Due to the thermally fully developed assumption, the enthalpy term $(U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y})$ in the fluid energy equation is neglected. This implies that the momentum equation does not affect the energy equation but the energy equation has a strong effect on the momentum equation. A similar case is considered by Bejan [15] but for steady free convection in a clear (non-porous) domain using the one-temperature model.

Also, it is clear from Eqs. (1) and (4) that the Brinkman term is added to extend the Darcy model. Including the Brinkman term improves the predictions of the Darcy model especially near the walls and within the hydrodynamic boundary layer. In this case, the model is able to satisfy the velocity no-slip condition at the wall.

3. SOLUTION METHODOLOGY

The harmonic fluctuations in the imposed wall temperatures are the only source for the disturbances in the thermal and hydrodynamic behaviors of the channel flow. The nature of these harmonic fluctuations implies that U, θ_f, and θ_s vary in the form:

$$\begin{aligned} U(\tau, Y) &= \mathrm{Im}\left\{W(Y)\, e^{i\omega\tau}\right\} \\ \theta_f(\tau, Y) &= \mathrm{Im}\left\{V_f(Y)\, e^{i\omega\tau}\right\} \\ \theta_s(\tau, Y) &= \mathrm{Im}\left\{V_s(Y)\, e^{i\omega\tau}\right\} \end{aligned} \tag{8}$$

Under this assumption, the governing equations are reduced to

- Case I:

$$-C_a\, i\omega\, W + \frac{\partial^2 W}{\partial Y^2} - \frac{W}{Da} = V_f \tag{9}$$

$$i\omega\, V_s = \frac{\partial^2 V_s}{\partial Y^2} + H(V_f - V_s) \tag{10}$$

$$i\omega\, V_f = C\, H(V_s - V_f) \tag{11}$$

- Case II:

$$-C_a\, i\omega\, W + \frac{\partial^2 W}{\partial Y^2} - \frac{W}{Da} = V_f \tag{12}$$

$$i\omega\, V_s = H(V_f - V_s) \tag{13}$$

$$\alpha\, i\omega\, V_f = \frac{\partial^2 V_f}{\partial Y^2} + r_k H(V_s - V_f) \tag{14}$$

The solutions for these governing equations are given as

- Case I:

$$V_s(Y) = \beta \frac{\cosh \lambda Y}{\cosh \lambda} \tag{15}$$

$$V_f(Y) = \gamma\, V_s(Y) \tag{16}$$

$$W(Y) = F\cosh\left(\frac{Y}{\sqrt{Da}}\right) + \beta\gamma\frac{\cosh\lambda Y}{(\cosh\lambda)\left(\lambda^2 - 1/Da\right)} \tag{17}$$

where

$$\lambda = \sqrt{H + i\omega - H\gamma}, \quad \gamma = \frac{CH}{i\omega + CH},$$
$$F = \frac{-\beta\gamma}{\left(\lambda^2 - 1/Da\right)\cosh(1\sqrt{Da})} \tag{18}$$

- Case II:

$$V_f(Y) = \beta\frac{\cosh\lambda Y}{\cosh\lambda} \tag{19}$$

$$V_s(Y) = \gamma\, V_f(Y) \tag{20}$$

$$W(Y) = F\cosh\left(\frac{Y}{\sqrt{Da}}\right) + \beta\frac{\cosh\lambda Y}{(\cosh\lambda)\left(\lambda^2 - 1/Da\right)} \tag{21}$$

where

$$\lambda = \sqrt{H r_k - H\gamma r_k + i\omega\alpha}, \quad \gamma = \frac{H}{H + i\omega},$$
$$F = \frac{-\beta}{\left(\lambda^2 - 1/Da\right)\cosh(1/\sqrt{Da})} \tag{22}$$

The final solutions for U, θ_f and θ_s of both cases may be obtained by inserting Eqs. (15) to (17) into Eq. (8) for Case I and inserting Eqs. (19) to (21) into Eq. (8) for Case II. The final step in the solution is to estimate the imaginary part of $W(y)e^{i\omega\tau}$, $V_f(y)e^{i\omega\tau}$, and $V_s(y)e^{i\omega\tau}$. This may be accomplished using any of the commercial packages such as MatLab or Mathematica.

The local thermal equilibrium criterion comparing V_f and V_s in Eqs. (16) and (20) reveals that the conditions of thermal equilibrium should hold if the following criterion is satisfied:

$$\gamma \approx 1 \tag{23}$$

This implies, within 5% error, that the criteria for local equilibrium are reduced to

- Case I:

$$\frac{\omega}{CH} < 0.05 \tag{24}$$

- Case II:

$$\frac{\omega}{H} < 0.05 \tag{25}$$

The criteria, Eqs. (24) and (25), imply that an increase of the frequency of the fluctuations in the wall temperature restricts the validity of the thermal equilibrium assumption. It is obvious that the ability of the solid matrix to sense the thermal fluctuations in the fluid temperature becomes weak as the frequency of these fluctuations increases. Also, the validity of the thermal equilibrium assumption is demonstrated as the volumetric Nusselt number increases. As the convective heat transfer between the fluid and solid domains increases, the time required by the solid domain to attain the fluid temperature decreases. Also, it is clear from the criterion in Eq. (24) that for Case I, the thermal equilibrium assumption is established for large values of C. In Case I, the transverse conduction in the fluid domain is neglected which implies that the effect of the thermal disturbance is carried into the channel directly through the solid matrix and then the solid matrix transfers it to the fluid domain through the volumetric convective heat transfer coefficient. Now, very large values of C imply that the fluid domain has low total thermal capacity $\varepsilon \rho_f c_f$ and the solid domain has high total thermal capacity $(1-\varepsilon)\rho_s c_s$. This means that the solid domain, which carries the effect of the thermal disturbance, is able to transfer a small part of its energy to the fluid domain, which is sufficient to raise the fluid temperature so both domains attain approximately equal temperatures. This confirms the local thermal equilibrium assumption. On the other hand, if C is very low, this implies that the fluid domain needs a large amount of energy to attain a temperature similar to the solid domain. This condition needs more time and may not be supported.

4. RESULTS AND DISCUSSION

Due to the fully developed thermal equilibrium assumption, the momentum equation does not affect the energy equation but the energy equation has a strong effect on the momentum equation. This implies that all parameters appear in the momentum equations (Eqs. (1) and (4)), i.e., C_a and Da, and do not affect the local thermal equilibrium assumption. These two parameters have a significant effect on the channel hydrodynamic behavior as is clear from the solutions given for U.

The effects of different parameters on the validity of the local thermal equilibrium assumption are investigated in Figs. 2–11 for both Cases I and II. Figures 2 and 3 show the effect of the volumetric H on the absolute difference between the solid and fluid temperatures $|\theta_s - \theta_f|$. As predicted, this difference decreases as H increases. An increase of H enhances the heat transfer between the fluid and the solid matrix, and this in turn shortens the time required to attain local thermal equilibrium. In Figs. 2 and 3 the amplitude of the thermal distribution is $\beta = 10$ and the deviation between the fluid and solid matrix temperature may reach *90%* of this value at very small values of H. Also, Figs. 2 and 3 show that the maximum deviation between θ_s and θ_f occurs at different times as H changes. This is predicted since the phase lag between θ_s and θ_f is very sensitive to H. Figure 4 shows the effect of C on the absolute temperature difference $|\theta_s - \theta_f|$ at different H. In Case I, the effect of the thermal disturbance is carried from the wall to the interior domain through the solid matrix. Large values of C imply that the solid domain has higher total thermal capacity $(1-\varepsilon)\,\rho_s c_s$ as compared to the fluid one $\varepsilon\,\rho_f c_f$. This implies that the solid domain is able to raise the fluid domain temperature very easily so both domains attain approximately the same temperatures. This behavior is justified previously in the section describing the thermal equilibrium criterion.

Figure 5 shows the effect of r_k on the temperature difference for Case II at different values of H and α. From Eq. (6) it is clear that r_k has the same effect as H on the validity of the local thermal equilibrium, but

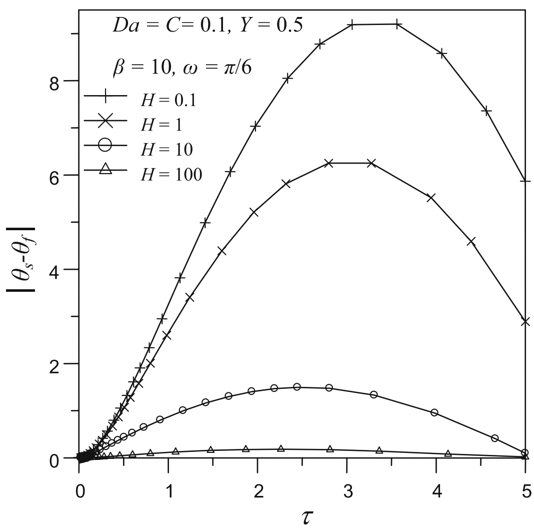

Fig. 2. Transient behavior of the difference between the fluid and solid temperatures at different H numbers; Case I.

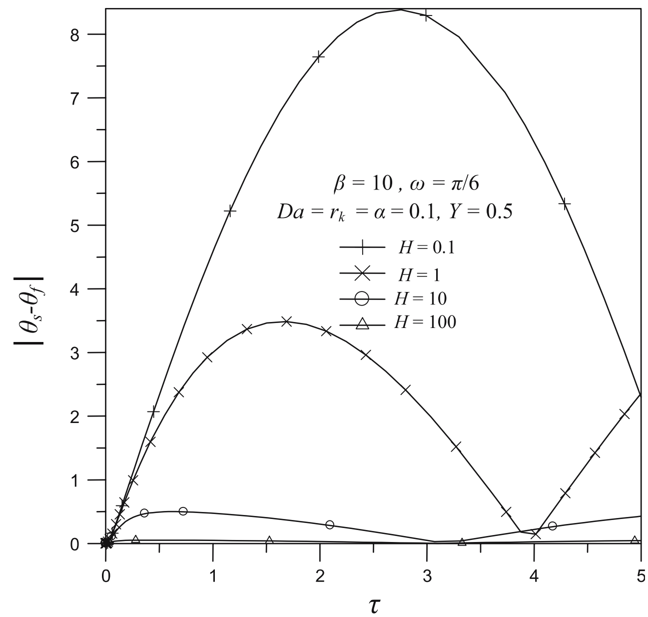

Fig. 3. Transient behavior of the difference between the fluid and solid temperatures at different H numbers; Case II.

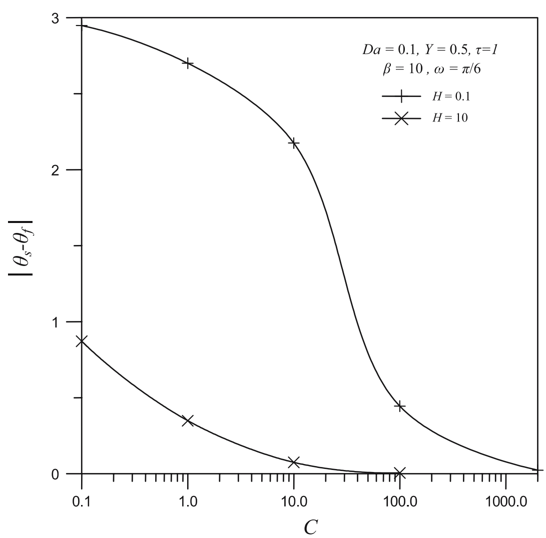

Fig. 4. Effect of C on the difference between the fluid and solid temperatures at different H numbers; Case I.

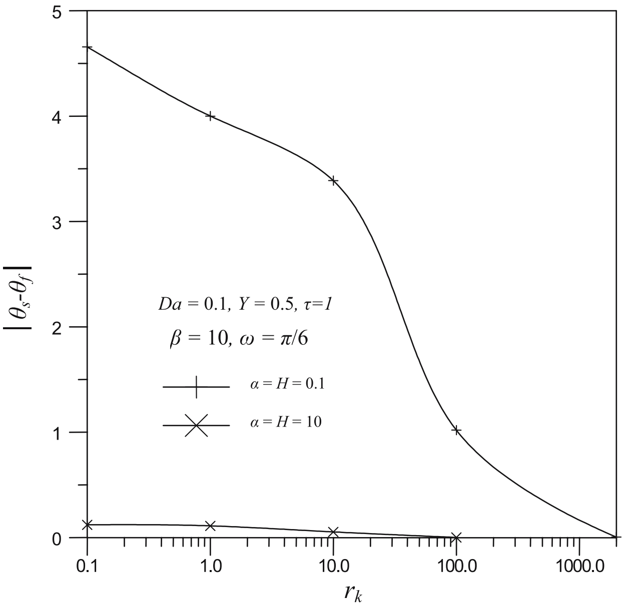

Fig. 5. Effect of r_k on the difference between the fluid and solid temperatures at different H numbers; Case II.

the r_k effect is not as strong as the$H$ effect as will be shown later. An increase of the coefficient of $(\theta_s - \theta_f)$ in Eq. (6) demonstrates the validity of the local thermal equilibrium assumption. This implies that an increase of r_k will support the local thermal equilibrium assumption. It is clear from Fig. 5 that the effect of r_k on the local thermal equilibrium assumption is more significant at small values of H. The effect of H and C on the temperature difference of Case I is shown from another point of view in Fig. 6. It is worth mentioning here that the H-axis is plotted on a log scale. This implies that the effect of the H number on the temperature difference is insignificant at large values of H. This is justified, since the time required for both fluid and solid domains to attain the same temperature is proportional to $1/q$, where q is the convective heat transfer between the fluid and solid domains. This convective heat transfer is proportional to h which is the volumetric convective heat transfer coefficient. As a result, the temperature difference $|\theta_s - \theta_f|$ is proportional to $1/h$ or to $1/H$.

The effect of H on $|\theta_s - \theta_f|$ at different α and r_k is shown in Fig. 7 for Case II from another point of view. The behavior shown on this figure is explained previously. The effect of α on the temperature difference

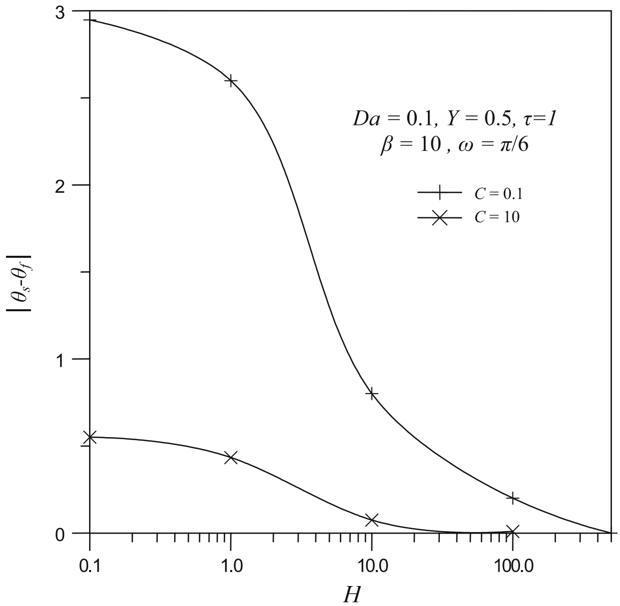

Fig. 6. Effect of H number on the difference between the fluid and solid temperatures at different C; Case I.

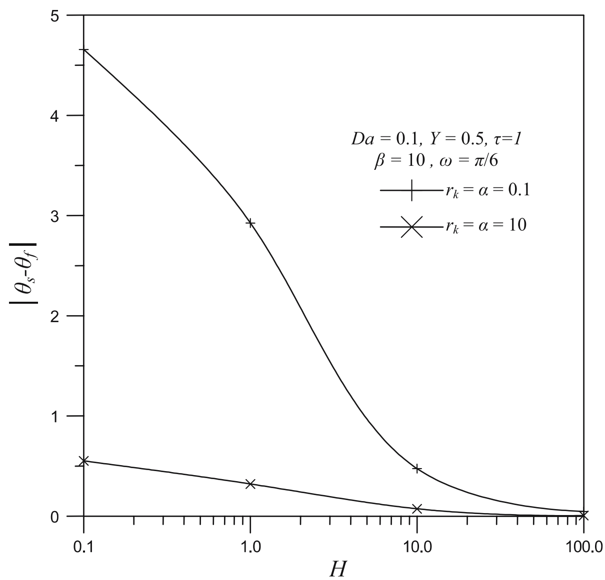

Fig. 7. Effect of H number on the difference between the fluid and solid temperatures at different r_k and α; Case II.

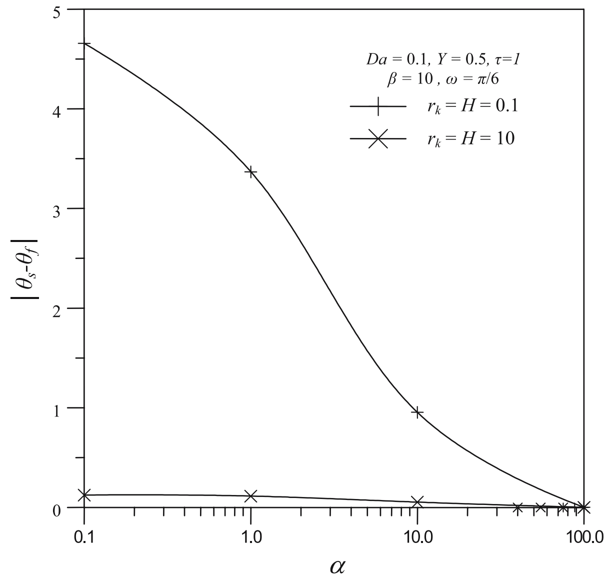

Fig. 8. Effect of α on the difference between the fluid and solid temperatures at different H and r_k; Case II.

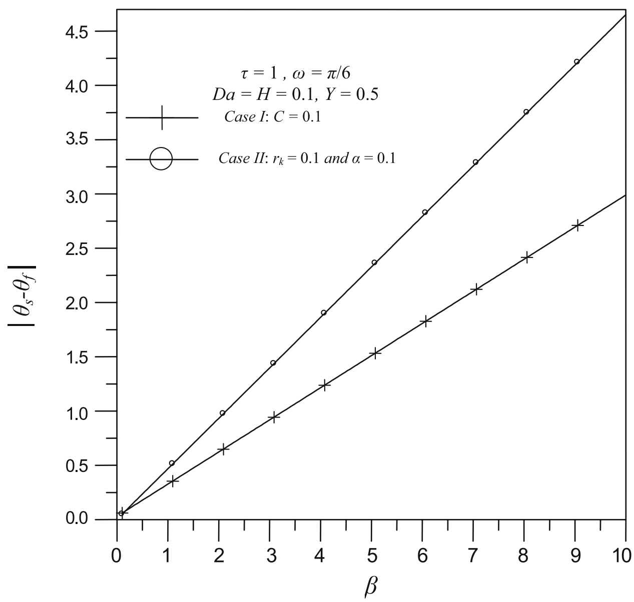

Fig. 9. Effect of β on the difference between the fluid and solid temperatures; Case I and Case II.

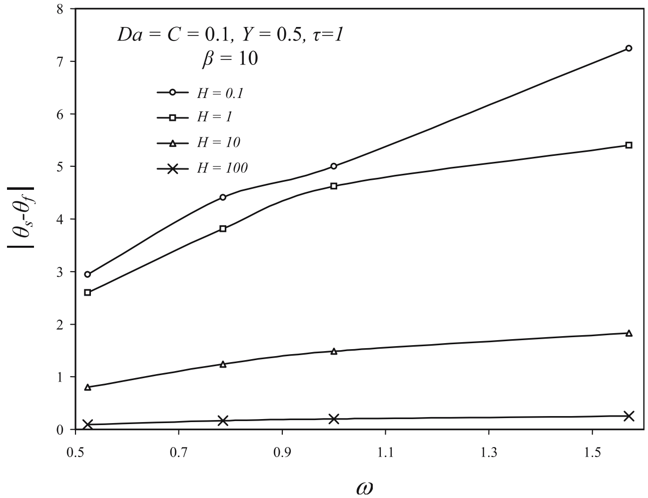

Fig. 10. Effect of ω on the difference between the fluid and solid temperatures at different H numbers; Case I.

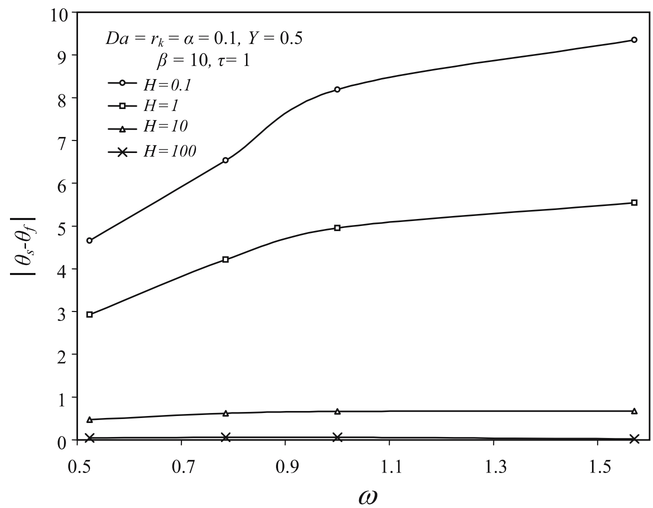

Fig. 11. Effect of ω on the difference between the fluid and solid temperatures at different H numbers; Case II.

for Case II is shown in Fig. 8. The temperature difference decreases as α increases. The effect of α on the temperature difference is more significant at small values of H and r_k.

The effect of the amplitude of the thermal disturbance β on the temperature difference is shown in Fig. 9 for Cases I and II. As predicted, the temperature difference is linearly proportional to the amplitude β. This is obvious since the energy equations are linear and homogeneous, and the thermal boundary conditions are either homogeneous or linearly proportional to β. As a result, it is predicted that θ_f and θ_s and the difference $|\theta_s - \theta_f|$ are linearly proportional to β. The local thermal equilibrium assumption is demonstrated in applications involving a weak thermal disturbance.

The effect of the dimensionless thermal disturbance frequency ω on the temperature difference is shown in Figs. 10 and 11 for Cases I and II, respectively. The temperature difference $|\theta_s - \theta_f|$ increases as ω increases for both cases. The ability of one domain to sense the thermal fluctuations carried by the other domain becomes weak as the frequency of these fluctuations increases. The effect of ω on the temperature difference is more significant at small values of H. Also, the effect of ω on the temperature difference is less significant at large values of ω.

5. CONCLUSIONS

The Darcy–Brinkman model is used to investigate the local thermal equilibrium assumption in the periodic free convection porous channel. It is found that the volumetric Nusselt number, thermal conductivity ratio, natural frequency ratio, and the amplitude have the most significant effect on the local thermal equilibrium assumption. The local thermal equilibrium assumption is demonstrated for large values of the H number, thermal conductivity ratio, and thermal diffusivity ratio and for small values of amplitude and frequency of the thermal disturbance.

NOMENCLATURE

c	Specific thermal capacity
C	Total thermal capacity ratio, $(1-\varepsilon)\, c_s \rho_s / \left(\varepsilon \rho_f c_f\right)$
c_a	Acceleration coefficient tensor
C_a	Modified acceleration coefficient tensor, $c_a \alpha_s \rho_f / \mu_{\text{eff}}$
Da	Darcy number, $(K^* \mu_{\text{eff}})/(L^2 \mu_f)$
h	Volumetric heat transfer coefficient
H	Volumetric Nusselt number, $h L^2/((1-\varepsilon)k_s)$
k	Thermal conductivity
K^*	Permeability
k_f	Thermal conductivity of the fluid
k_s	Thermal conductivity of the solid
$2L$	Channel width
r_k	Thermal conductivity ratio, $(1-\varepsilon)\, k_s / \left(\varepsilon k_f\right)$
t	Time
t_o	Reference time, L^2/α_s
T	Temperature
T_w	Wall temperature
T_∞	Ambient temperature
u	Axial velocity
U	Dimensionless axial velocity, u/u_o
u_o	Reference velocity, $\left(\rho_f L^2 g \bar{\beta} \Delta T\right) \mu_{\text{eff}}$
x	Transverse coordinate
X	Dimensionless transverse coordinate, x/L
y	Axial coordinate
Y	Dimensionless axial coordinate, y/L
Greek symbols	
α	Thermal diffusivity ratio, α_s/α_f
α_f	Thermal diffusivity of the fluid
α_s	Thermal diffusivity of the solid
β	Amplitude of the thermal disturbance, $T_w/(T_w - T_\infty)$
$\bar{\beta}$	Coefficient of thermal expansion
ΔT	Temperature difference, $T_w - T_\infty$
ε	Porosity
μ_f	Dynamic viscosity of the fluid
μ_{eff}	Effective dynamic viscosity
θ	Dimensionless temperature, $(T - T_\infty)/(T_w - T_\infty)$
θ_f	Dimensionless fluid temperature, $\left(T_f - T_\infty\right)/(T_w - T_\infty)$
θ_s	Dimensionless solid temperature, $(T_s - T_\infty)/(T_w - T_\infty)$
τ	Dimensionless time, t/t_o
ω	Dimensionless frequency of the thermal disturbance, $\overline{\omega} L/\alpha_s$
$\overline{\omega}$	Frequency of the thermal disturbance

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